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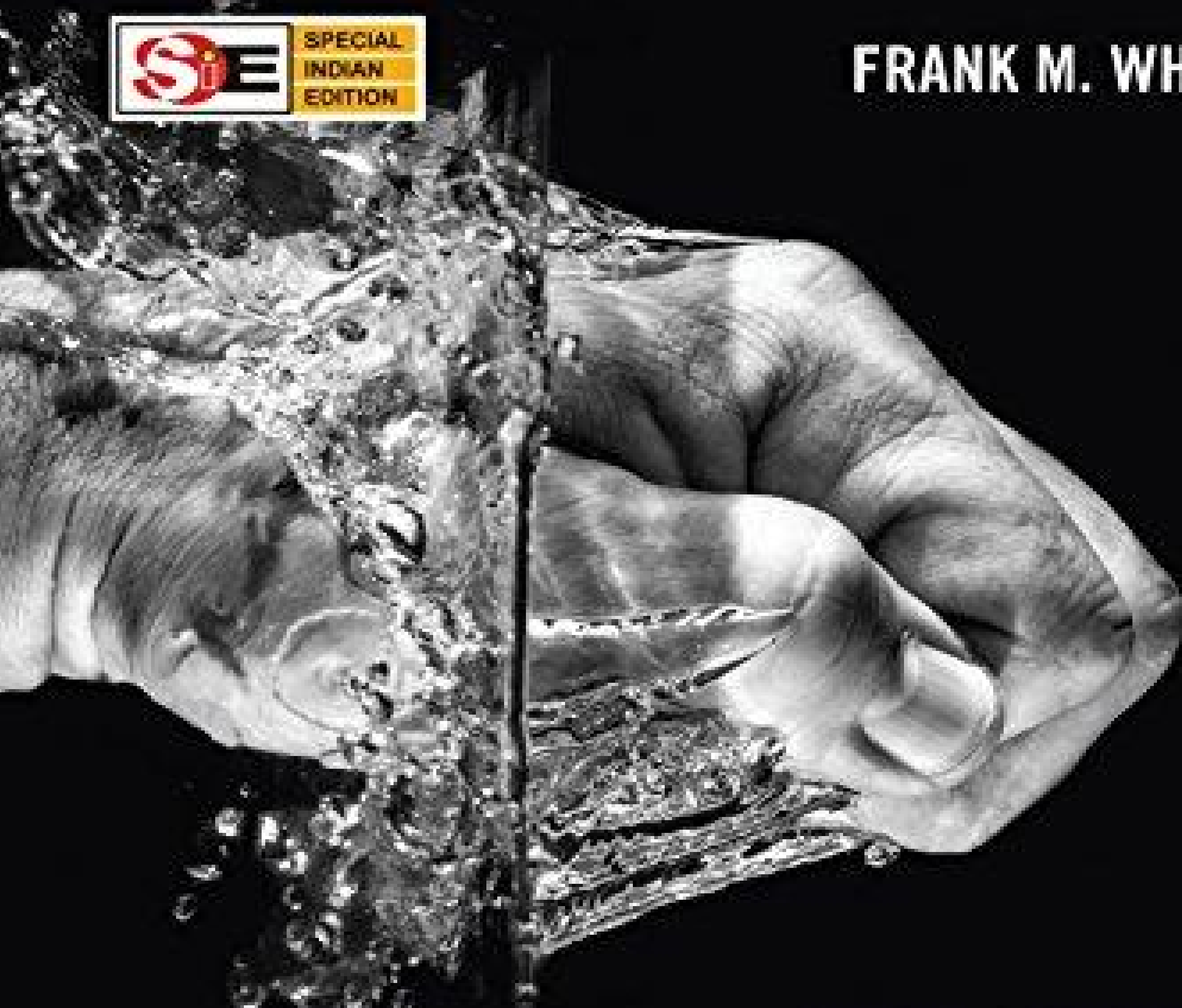
FRANK M. WHITE



FLUID MECHANICS

8TH EDITION IN SI UNITS

McGraw Hill Education

Fluid Mechanics

Fluid Mechanics

Eighth Edition

Frank M. White
University of Rhode Island

McGraw Hill Education

FLUID MECHANICS, EIGHTH EDITION

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About the Author

Frank M. White is Professor Emeritus of Mechanical and Ocean Engineering at the University of Rhode Island. He studied at Georgia Tech and M.I.T. In 1966 he helped found, at URI, the first department of ocean engineering in the country. Known primarily as a teacher and writer, he has received eight teaching awards and has written four textbooks on fluid mechanics and heat transfer.

From 1979 to 1990, he was editor-in-chief of the *ASME Journal of Fluids Engineering* and then served from 1991 to 1997 as chairman of the ASME Board of Editors and of the Publications Committee. He is a Fellow of ASME and in 1991 received the ASME Fluids Engineering Award. He lives with his wife, Jeanne, in Narragansett, Rhode Island.

To Jeanne

Contents

Preface

General Approach

The eighth edition of *Fluid Mechanics* sees some additions and deletions but no philosophical change. The basic outline of eleven chapters, plus appendices, remains the same. The triad of integral, differential, and experimental approaches is retained. Many problem exercises, and some fully worked examples, have been changed. The informal, student-oriented style is retained. A number of new photographs and figures have been added. Many new references have been added, for a total of 445. The writer is a firm believer in "further reading," especially in the postgraduate years.

Learning Tools

The total number of problem exercises continues to increase, from 1089 in the first edition, to 1683 in this eighth edition. There are approximately 20 new problems in each chapter. Most of these are basic end-of-chapter problems, classified according to topic. There are also Word Problems, multiple-choice Fundamentals of Engineering Problems, Comprehensive Problems, and Design Projects. The appendix lists approximately 700 Answers to Selected Problems.

The example problems are structured in the text to follow the sequence of recommended steps outlined in Section 1.7.

Most of the problems in this text can be solved with a hand calculator. Some can even be simply explained in words. A few problems, especially in Chapters 6, 9, and 10, involve solving complicated algebraic expressions, laborious for a hand calculator. Check to see if your institution has a license for equation-solving software. Here the writer solves complicated example problems by using the iterative power of Microsoft Office Excel, as illustrated, for example, in Example 6.5. For further use in your work, Excel also contains several hundred special mathematical functions for engineering and statistics. Another benefit: Excel is free.

Content Changes

There are some revisions in each chapter.

Chapter 1 has been substantially revised. The pre-reviewers felt, correctly, that it was too long, too detailed, and at too high a level for an introduction. Former Section 1.2, History of Fluid Mechanics, has been shortened and moved to the end of the chapter. Former Section 1.3, Problem-Solving Techniques, has been moved to appear just before Example 1.7, where these techniques are first used. Eulerian and Lagrangian descriptions have been moved to Chapter 4. A temperature-entropy chart for steam has been added, to illustrate when steam can and cannot be approximated as an ideal gas. Former Section 1.11, Flow Patterns, has been cut sharply and mostly moved to Chapter 4. Former Section 1.13, Uncertainty in Experimental Data, has been moved to a new Appendix E. No one teaches "uncertainty" in introductory fluid mechanics, but the writer feels it is extremely important in all engineering fields involving experimental or numerical data.

Chapter 2 adds a brief discussion of the fact that pressure is a thermodynamic property, not a *force*, has no direction, and is not a vector. The arrow, on a surface force caused by pressure, causes confusion for beginning students. The subsection of Section 2.8 entitled Stability Related to Waterline Area has been shortened to omit the complicated derivations. The final metacenter formula is retained; the writer does not think it is sufficient just to show a sketch of a floating body falling over. This book should have reference value.

Chapter 3 was substantially revised in the last edition, especially by moving Bernoulli's equation to follow the linear momentum section. This time the only changes are improvements in the example problems.

Chapter 4 now discusses the Eulerian and Lagrangian systems, moved from Chapter 1. The no-slip and no-temperature-jump boundary conditions are added, with problem assignments.

Chapter 5 explains a bit more about drag force before assigning dimensional analysis problems. It retains Ipsen's method as an interesting alternative which, of course, may be skipped by pi theorem adherents.

Chapter 6 downplays the Moody chart a bit, suggesting that students use either iteration or Excel. For rough walls, the chart is awkward to read, although it gives an approximation for use in iteration. The author's fancy rearrangement of pi groups to solve type 2, flow rate, and type 3, pipe diameter problems is removed from the main text and assigned as problems. For noncircular ducts, the hydraulic *radius* is omitted and moved to Chapter 10. There is a new Example 6.11, which solves for pipe diameter and determines if Schedule 40 pipe is strong enough. A general discussion of pipe strength is added. There is a new subsection on *laminar-flow* minor losses, appropriate for micro- and nano-tube flows.

Chapter 7 has more treatment of vehicle drag and rolling resistance, and a rolling resistance coefficient is defined. There is additional discussion of the Kline-Fogelman airfoil, extremely popular now for model aircraft.

Chapter 8 has backed off from extensive discussion of CFD methods, as proposed by the pre-reviewers. Only a few CFD examples are now given. The inviscid duct-expansion example and the implicit boundary layer method are now omitted, but the explicit method is retained. For airfoil theory, the writer considers thin-airfoil vortex-sheet theory to be obsolete and has deleted it.

Chapter 9 now has a better discussion of the normal shock wave. New supersonic wave photographs are added. The "new trend in aeronautics" is the Air Force X-35 Joint Strike Fighter.

Chapter 10 improves the definition of normal depth of a channel. There is a new subsection on the water-channel compressible flow analogy, and problems are assigned to find the oblique wave angle for supercritical water flow past a wedge.

Chapter 11 greatly expands the discussion of wind turbines, with examples and problems taken from the author's own experience.

Appendices B and D are unchanged. Appendix A adds a list of liquid kinematic viscosities to Table A.4. A few more conversion factors are added to Appendix C. There is a new Appendix E, Estimating Uncertainty in Experimental Data, which was moved from its inappropriate position in Chapter 1. The writer believes that "uncertainty" is vital to reporting measurements and always insisted upon it when he was an engineering journal editor.

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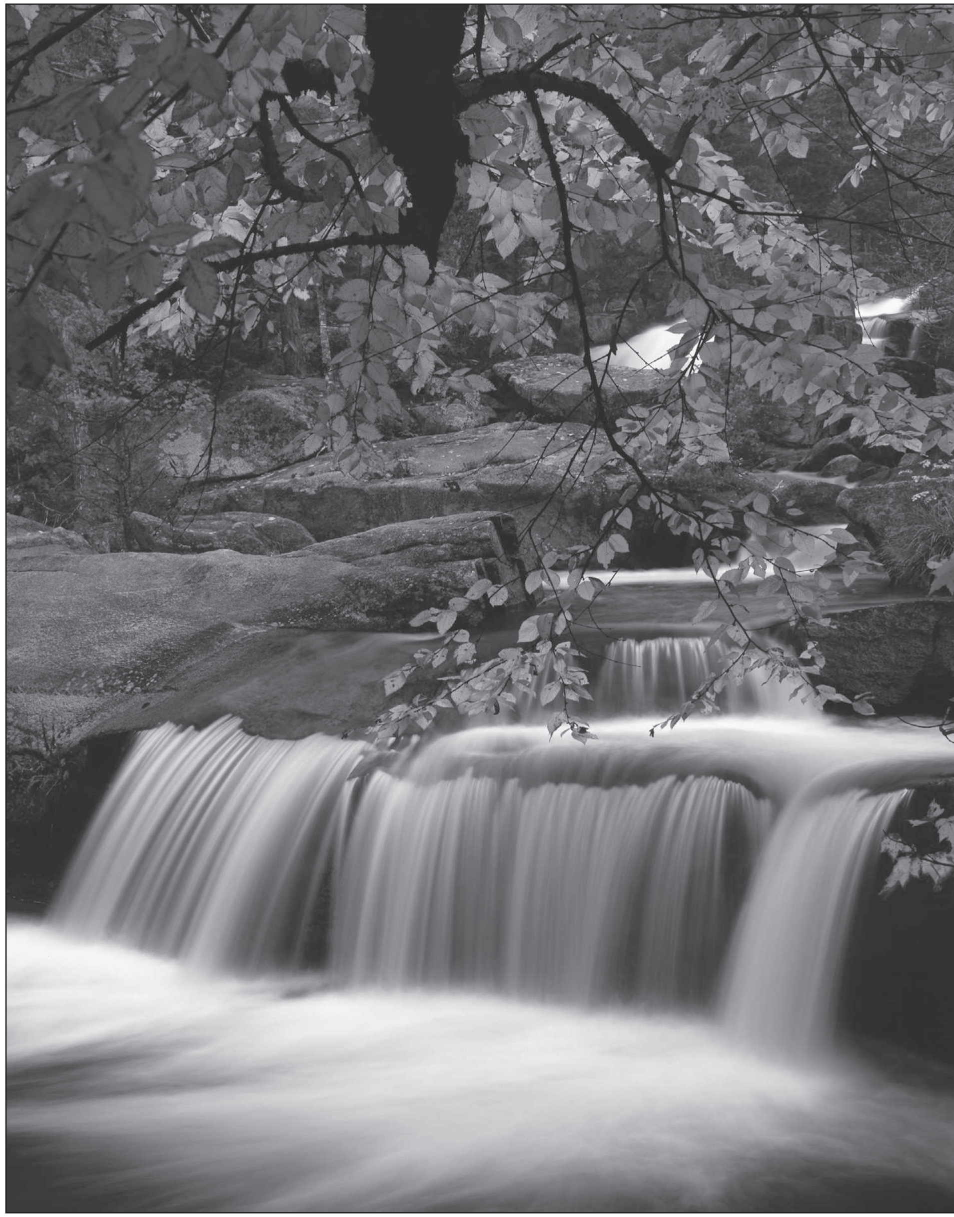
Finally, I am thankful for the continuing support of my family, especially Jeanne, who remains in my heart, and my sister Sally White GNSH, my dog Jack, and my cats Cole and Kerry.

Fluid Mechanics



Falls on the Nesowadnehunk Stream in Baxter State Park, Maine, which is the northern terminus of the Appalachian Trail. Such flows, open to the atmosphere, are driven simply by gravity and do not depend much upon fluid properties such as density and viscosity. They are discussed later in Chap. 10. To the writer, one of the joys of fluid mechanics is that visualization of a fluid-flow process is simple and beautiful [*Photo Credit: Design Pics/Natural Selection Robert Cable*].

Chapter 1
Introduction

1.1 Preliminary Remarks

Fluid mechanics is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*). Both gases and liquids are classified as fluids, and the number of fluid engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few. When you think about it, almost everything on this planet either is a fluid or moves within or near a fluid.

The essence of the subject of fluid flow is a judicious compromise between theory and experiment. Since fluid flow is a branch of mechanics, it satisfies a set of well-documented basic laws, and thus a great deal of theoretical treatment is available. However, the theory is often frustrating because it applies mainly to idealized situations, which may be invalid in practical problems. The two chief obstacles to a workable theory are geometry and viscosity. The basic equations of fluid motion (Chap. 4) are too difficult to enable the analyst to attack arbitrary geometric configurations. Thus most textbooks concentrate on flat plates, circular pipes, and other easy geometries. It is possible to apply numerical computer techniques to complex geometries, and specialized textbooks are now available to explain the new *computational fluid dynamics* (CFD) approximations and methods [1–4].[1] This book will present many theoretical results while keeping their limitations in mind.

The second obstacle to a workable theory is the action of viscosity, which can be neglected only in certain idealized flows (Chap. 8). First, viscosity increases the difficulty of the basic equations, although the boundary-layer approximation found by Ludwig Prandtl in 1904 (Chap. 7) has greatly simplified viscous-flow analyses. Second, viscosity has a destabilizing effect on all fluids, giving rise, at frustratingly small velocities, to a disorderly, random phenomenon called *turbulence.* The theory of turbulent flow is crude and heavily backed up by experiment (Chap. 6), yet it can be quite serviceable as an engineering estimate. This textbook only introduces the standard experimental correlations for turbulent time-mean flow. Meanwhile, there are advanced texts on both time-mean *turbulence and turbulence modeling* [5, 6] and on the newer, computer-intensive *direct numerical simulation* (DNS) of fluctuating turbulence [7, 8].

[1]Numbered references appear at the end of each chapter.

Thus there is theory available for fluid flow problems, but in all cases it should be backed up by experiment. Often the experimental data provide the main source of information about specific flows, such as the drag and lift of immersed bodies (Chap. 7). Fortunately, fluid mechanics is a highly visual subject, with good instrumentation [9–11], and the use of dimensional analysis and modeling concepts (Chap. 5) is widespread. Thus experimentation provides a natural and easy complement to the theory. You should keep in mind that theory and experiment should go hand in hand in all studies of fluid mechanics.

1.2 The Concept of a Fluid

From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. The difference between the two is perfectly obvious to the layperson, and it is an interesting exercise to ask a layperson to put this difference into words. The technical distinction lies with the reaction of the two to an applied shear or tangential stress. *A solid can resist a shear stress by a static deflection; a fluid cannot*. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. As a corollary, we can say that a fluid at rest must be in a state of zero shear stress, a state often called the hydrostatic stress condition in structural analysis. In this condition, Mohr's circle for stress reduces to a point, and there is no shear stress on any plane cut through the element under stress.

Given this definition of a fluid, every layperson also knows that there are two classes of fluids, *liquids* and *gases.* Again the distinction is a technical one concerning the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above. Free-surface flows are dominated by gravitational effects and are studied in Chaps. 5 and 10. Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand until it encounters confining walls. A gas has no definite volume, and when left to itself without confinement, a gas forms an atmosphere that is essentially hydrostatic. The hydrostatic behavior of liquids and gases is taken up in Chap. 2. Gases cannot form a free surface, and thus gas flows are rarely concerned with gravitational effects other than buoyancy.

Figure 1.1 illustrates a solid block resting on a rigid plane and stressed by its own weight. The solid sags into a static deflection, shown as a highly exaggerated dashed line, resisting shear without flow. A free-body diagram of element A on the side of the block shows that there is shear in the block along a plane cut at an angle θ through A. Since the block sides are unsupported, element A has zero stress on the left and right sides and compression stress $\sigma = -p$ on the top and bottom. Mohr's circle does not reduce to a point, and there is nonzero shear stress in the block.

By contrast, the liquid and gas at rest in Fig. 1.1 require the supporting walls in order to eliminate shear stress. The walls exert a compression stress of $-p$ and reduce Mohr's circle to a point with zero shear everywhere—that is, the hydrostatic condition. The liquid retains its volume and forms a free surface in the container. If the walls are removed, shear develops in the liquid and a big splash results. If the container is tilted, shear again develops, waves form, and the free surface seeks a horizontal configuration, pouring out over the lip if necessary. Meanwhile, the gas is unrestrained

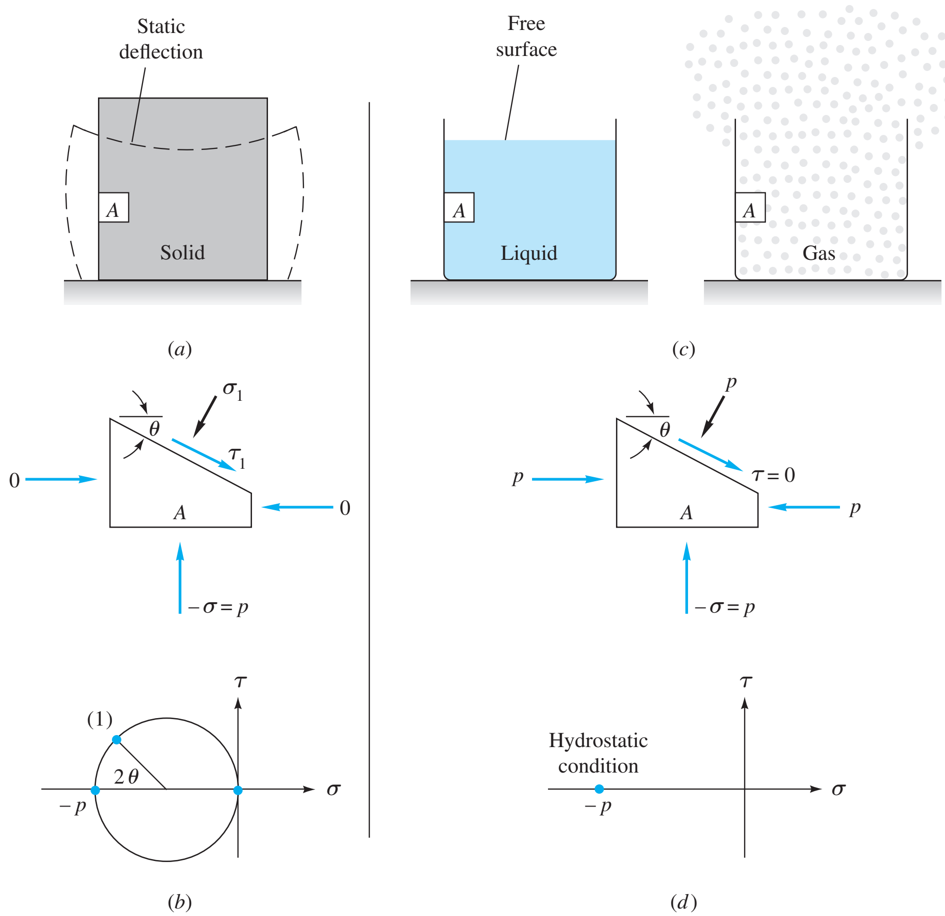


Fig. 1.1 A solid at rest can resist shear. (*a*) Static deflection of the solid; (*b*) equilibrium and Mohr's circle for solid element *A*. A fluid cannot resist shear. (*c*) Containing walls are needed; (*d*) equilibrium and Mohr's circle for fluid element *A*.

and expands out of the container, filling all available space. Element *A* in the gas is also hydrostatic and exerts a compression stress $-p$ on the walls.

In the previous discussion, clear decisions could be made about solids, liquids, and gases. Most engineering fluid mechanics problems deal with these clear cases—that is, the common liquids, such as water, oil, mercury, gasoline, and alcohol, and the common gases, such as air, helium, hydrogen, and steam, in their common temperature and pressure ranges. There are many borderline cases, however, of which you should be aware. Some apparently "solid" substances such as asphalt and lead resist shear stress for short periods but actually deform slowly and exhibit definite fluid behavior over long periods. Other substances, notably colloid and slurry mixtures, resist small shear stresses but "yield" at large stress and begin to flow as fluids do. Specialized textbooks are devoted to this study of more general deformation and flow, a field called *rheology* [16]. Also, liquids and gases can coexist in two-phase mixtures, such as steam–water mixtures or water with entrapped air bubbles. Specialized textbooks present the analysis of such *multiphase flows* [17]. Finally, in some situations the distinction between a liquid and a gas blurs. This is the case at temperatures and pressures above the so-called *critical point* of a substance, where only a single phase exists, primarily resembling a gas. As pressure increases far above the critical point, the gaslike substance becomes so dense that there is some resemblance to a liquid, and the usual thermodynamic approximations like the perfect-gas law become inaccurate. The critical temperature and pressure of water are $T_c = 647$ K and $p_c = 219$ atm (atmosphere)[2] so that typical problems involving water and steam are below the critical point. Air, being a mixture of gases, has no distinct critical point, but its principal component, nitrogen, has $T_c = 126$ K and $p_c = 34$ atm. Thus typical problems involving air are in the range of high temperature and low pressure where air is distinctly and definitely a gas. This text will be concerned solely with clearly identifiable liquids and gases, and the borderline cases just discussed will be beyond our scope.

1.3 The Fluid as a Continuum

We have already used technical terms such as *fluid pressure* and *density* without a rigorous discussion of their definition. As far as we know, fluids are aggregations of molecules, widely spaced for a gas, closely spaced for a liquid. The distance between molecules is very large compared with the molecular diameter. The molecules are not fixed in a lattice but move about freely relative to each other. Thus fluid density, or mass per unit volume, has no precise meaning because the number of molecules occupying a given volume continually changes. This effect becomes unimportant if the unit volume is large compared with, say, the cube of the molecular spacing, when the number of molecules within the volume will remain nearly constant in spite of the enormous interchange of particles across the boundaries. If, however, the chosen unit volume is too large, there could be a noticeable variation in the bulk aggregation of the particles. This situation is illustrated in Fig. 1.2, where the "density" as calculated from molecular mass δm within a given volume $\delta\mathcal{V}$ is plotted versus the size of the unit volume. There is a limiting volume $\delta\mathcal{V}^*$ below which molecular variations may be important and above which aggregate variations may be important. The *density* ρ of a fluid is best defined as

$$\rho = \lim_{\delta\mathcal{V} \to \delta\mathcal{V}^*} \frac{\delta m}{\delta\mathcal{V}} \tag{1.1}$$

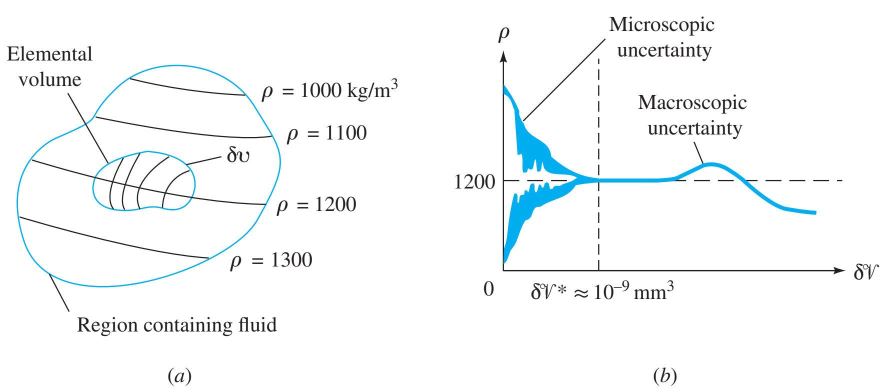


Fig. 1.2 The limit definition of continuum fluid density: (*a*) an elemental volume in a fluid region of variable continuum density; (*b*) calculated density versus size of the elemental volume.

[2]One atmosphere equals 2116 lbf/ft^2 = 101,300 Pa.

The limiting volume $\delta\mathcal{V}^*$ is about 10^{-9} mm^3 for all liquids and for gases at atmospheric pressure. For example, 10^{-9} mm^3 of air at standard conditions contains approximately 3×10^7 molecules, which is sufficient to define a nearly constant density according to Eq. (1.1). Most engineering problems are concerned with physical dimensions much larger than this limiting volume, so that density is essentially a point function and fluid properties can be thought of as varying continually in space, as sketched in Fig. 1.2a. Such a fluid is called a *continuum,* which simply means that its variation in properties is so smooth that differential calculus can be used to analyze the substance. We shall assume that continuum calculus is valid for all the analyses in this book. Again there are borderline cases for gases at such low pressures that molecular spacing and mean free path[3] are comparable to, or larger than, the physical size of the system. This requires that the continuum approximation be dropped in favor of a molecular theory of rarefied gas flow [18]. In principle, all fluid mechanics problems can be attacked from the molecular viewpoint, but no such attempt will be made here. Note that the use of continuum calculus does not preclude the possibility of discontinuous jumps in fluid properties across a free surface or fluid interface or across a shock wave in a compressible fluid (Chap. 9). Our calculus in analyzing fluid flow must be flexible enough to handle discontinuous boundary conditions.

1.4 Dimensions and Units

A *dimension* is the measure by which a physical variable is expressed quantitatively. A *unit* is a particular way of attaching a number to the quantitative dimension. Thus length is a dimension associated with such variables as distance, displacement, width, deflection, and height, while centimeters and inches are both numerical units for expressing length. Dimension is a powerful concept about which a splendid tool called *dimensional analysis* has been developed (Chap. 5), while units are the numerical quantity that the customer wants as the final answer.

In 1872 an international meeting in France proposed a treaty called the Metric Convention, which was signed in 1875 by 17 countries including the United States. It was an improvement over British systems because its use of base 10 is the foundation of our number system, learned from childhood by all. Problems still remained because even the metric countries differed in their use of kiloponds instead of dynes or newtons, kilograms instead of grams, or calories instead of joules. To standardize the metric system, a General Conference of Weights and Measures, attended in 1960 by 40 countries, proposed the *International System of Units* (SI). We are now undergoing a painful period of transition to SI, an adjustment that may take many more years to complete. The professional societies have led the way. Since July 1, 1974, SI units have been required by all papers published by the American Society of Mechanical Engineers, and there is a textbook explaining the SI [19]. The present text will use SI units together with British gravitational (BG) units.

Primary Dimensions

In fluid mechanics there are only four *primary dimensions* from which all other dimensions can be derived: mass, length, time, and temperature.[4] These dimensions and

[3] The mean distance traveled by molecules between collisions (see Prob. P1.5).

[4] If electromagnetic effects are important, a fifth primary dimension must be included, electric current $\{I\}$, whose SI unit is the ampere (A).

Table 1.1 Primary Dimensions in SI and BG Systems

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

their units in both systems are given in Table 1.1. Note that the Kelvin unit uses no degree symbol. The braces around a symbol like $\{M\}$ mean "the dimension" of mass. All other variables in fluid mechanics can be expressed in terms of $\{M\}$, $\{L\}$, $\{T\}$, and $\{\Theta\}$. For example, acceleration has the dimensions $\{LT^{-2}\}$. The most crucial of these secondary dimensions is force, which is directly related to mass, length, and time by Newton's second law. Force equals the time rate of change of momentum or, for constant mass,

$$\mathbf{F} = m\mathbf{a} \tag{1.2}$$

From this we see that, dimensionally, $\{F\} = \{MLT^{-2}\}$.

The International System (SI)

The use of a constant of proportionality in Newton's law, Eq. (1.2), is avoided by defining the force unit exactly in terms of the other basic units. In the SI system, the basic units are newtons $\{F\}$, kilograms $\{M\}$, meters $\{L\}$, and seconds $\{T\}$. We define

$$1 \text{ newton of force} = 1 \text{ N} = 1 \text{ kg} \cdot 1 \text{ m/s}^2$$

The newton is a relatively small force, about the weight of an apple (0.225 lbf). In addition, the basic unit of temperature $\{\Theta\}$ in the SI system is the degree Kelvin, K. Use of these SI units (N, kg, m, s, K) will require no conversion factors in our equations.

The British Gravitational (BG) System

In the BG system also, a constant of proportionality in Eq. (1.2) is avoided by defining the force unit exactly in terms of the other basic units. In the BG system, the basic units are pound-force $\{F\}$, slugs $\{M\}$, feet $\{L\}$, and seconds $\{T\}$. We define

$$1 \text{ pound of force} = 1 \text{ lbf} = 1 \text{ slug} \cdot 1 \text{ ft/s}^2$$

One lbf $\approx$ 4.4482 N and approximates the weight of four apples. We will use the abbreviation *lbf* for pound-force and *lbm* for pound-mass. The slug is a rather hefty mass, equal to 32.174 lbm. The basic unit of temperature $\{\Theta\}$ in the BG system is the degree Rankine, °R. Recall that a temperature difference 1 K = 1.8°R. Use of these BG units (lbf, slug, ft, s, °R) will require no conversion factors in our equations.

Other Unit Systems

There are other unit systems still in use. At least one needs no proportionality constant: the CGS system (dyne, gram, cm, s, K). However, CGS units are too small for most applications (1 dyne = 10^{-5} N) and will not be used here.

Table 1.2 Secondary Dimensions in Fluid Mechanics

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1\ m^2 = 10.764\ ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1\ m^3 = 35.315\ ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1\ ft/s^2 = 0.3048\ m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1\ lbf/ft^2 = 47.88\ Pa$
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1\ s^{-1} = 1\ s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	$J = N \cdot m$	$ft \cdot lbf$	$1\ ft \cdot lbf = 1.3558\ J$
Power $\{ML^2T^{-3}\}$	W = J/s	$ft \cdot lbf/s$	$1\ ft \cdot lbf/s = 1.3558\ W$
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1\ slug/ft^3 = 515.4\ kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	$kg/(m \cdot s)$	$slugs/(ft \cdot s)$	$1\ slug/(ft \cdot s) = 47.88\ kg/(m \cdot s)$
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot °R)$	$1\ m^2/(s^2 \cdot K) = 5.980\ ft^2/(s^2 \cdot °R)$

In the USA, some still use the English Engineering system (lbf, lbm, ft, s, °R), where the basic mass unit is the *pound of mass*. Newton's law (1.2) must be rewritten:

$$\mathbf{F} = \frac{m\mathbf{a}}{g_c}, \quad \text{where} \quad g_c = 32.174 \frac{\text{ft} \cdot \text{lbm}}{\text{lbf} \cdot \text{s}^2} \tag{1.3}$$

The constant of proportionality, g_c, has both dimensions and a numerical value not equal to 1.0. The present text uses only the SI and BG systems and will not solve problems or examples in the English Engineering system. Because Americans still use them, a few problems in the text will be stated in truly awkward units: acres, gallons, ounces, or miles. Your assignment will be to convert these and solve in the SI or BG systems.

The Principle of Dimensional Homogeneity

In engineering and science, *all* equations must be *dimensionally homogeneous,* that is, each additive term in an equation must have the same dimensions. For example, take Bernoulli's incompressible equation, to be studied and used throughout this text:

$$p + \frac{1}{2}\rho V^2 + \rho g Z = \text{constant}$$

Each and every term in this equation *must* have dimensions of pressure $\{ML^{-1}T^{-2}\}$. We will examine the dimensional homogeneity of this equation in detail in Example 1.3.

A list of some important secondary variables in fluid mechanics, with dimensions derived as combinations of the four primary dimensions, is given in Table 1.2. A more complete list of conversion factors is given in App. C.

EXAMPLE 1.1

A body weighs 1000 lbf when exposed to a standard earth gravity $g = 32.174\ ft/s^2$. (*a*) What is its mass in kg? (*b*) What will the weight of this body be in N if it is exposed to the moon's standard acceleration $g_{moon} = 1.62\ m/s^2$? (*c*) How fast will the body accelerate if a net force of 400 lbf is applied to it on the moon or on the earth?

Solution

We need to find the (*a*) mass; (*b*) weight on the moon; and (*c*) acceleration of this body. This is a fairly simple example of conversion factors for differing unit systems. No property data is needed. The example is too low-level for a sketch.

Part (a) Newton's law (1.2) holds with known weight and gravitational acceleration. Solve for m:

$$F = W = 1000 \text{ lbf} = mg = (m)(32.174 \text{ ft/s}^2), \text{ or } m = \frac{1000 \text{ lbf}}{32.174 \text{ ft/s}^2} = 31.08 \text{ slugs}$$

Convert this to kilograms:

$$m = 31.08 \text{ slugs} = (31.08 \text{ slugs})(14.5939 \text{ kg/slug}) = 454 \text{ kg} \qquad \textit{Ans. (a)}$$

Part (b) The mass of the body remains 454 kg regardless of its location. Equation (1.2) applies with a new gravitational acceleration and hence a new weight:

$$F = W_{\text{moon}} = mg_{\text{moon}} = (454 \text{ kg})(1.62 \text{ m/s}^2) = 735 \text{ N} = 165 \text{ lbf} \qquad \textit{Ans. (b)}$$

Part (c) This part does not involve weight or gravity or location. It is simply an application of Newton's law with a known mass and known force:

$$F = 400 \text{ lbf} = ma = (31.08 \text{ slugs})\, a$$

Solve for

$$a = \frac{400 \text{ lbf}}{31.08 \text{ slugs}} = 12.87 \frac{\text{ft}}{\text{s}^2}\left(0.3048 \frac{\text{m}}{\text{ft}}\right) = 3.92 \frac{\text{m}}{\text{s}^2} \qquad \textit{Ans. (c)}$$

Comment (c): This acceleration would be the same on the earth or moon or anywhere.

Many data in the literature are reported in inconvenient or arcane units suitable only to some industry or specialty or country. The engineer should convert these data to the SI or BG system before using them. This requires the systematic application of conversion factors, as in the following example.

EXAMPLE 1.2

Industries involved in viscosity measurement [27, 29] continue to use the CGS system of units, since centimeters and grams yield convenient numbers for many fluids. The absolute viscosity (μ) unit is the *poise,* named after J. L. M. Poiseuille, a French physician who in 1840 performed pioneering experiments on water flow in pipes; 1 poise = 1 g/(cm-s). The kinematic viscosity (ν) unit is the *stokes,* named after G. G. Stokes, a British physicist who in 1845 helped develop the basic partial differential equations of fluid momentum; 1 stokes = 1 cm^2/s. Water at 20°C has $\mu \approx 0.01$ poise and also $\nu \approx 0.01$ stokes. Express these results in (*a*) SI and (*b*) BG units.

Solution

Part (a)

- *Approach:* Systematically change grams to kg or slugs and change centimeters to meters or feet.

- *Property values:* Given $\mu = 0.01$ g/(cm-s) and $\nu = 0.01$ cm^2/s.
- *Solution steps:* (*a*) For conversion to SI units,

$$\mu = 0.01 \frac{\mathrm{g}}{\mathrm{cm \cdot s}} = 0.01 \frac{\mathrm{g(1\ kg/1000\ g)}}{\mathrm{cm(0.01\ m/cm)s}} = 0.001 \frac{\mathrm{kg}}{\mathrm{m \cdot s}}$$

$$\nu = 0.01 \frac{\mathrm{cm^2}}{\mathrm{s}} = 0.01 \frac{\mathrm{cm^2(0.01\ m/cm)^2}}{\mathrm{s}} = 0.000001 \frac{\mathrm{m^2}}{\mathrm{s}} \qquad \textit{Ans. (a)}$$

Part (b)

- For conversion to BG units

$$\mu = 0.01 \frac{\mathrm{g}}{\mathrm{cm \cdot s}} = 0.01 \frac{\mathrm{g(1\ kg/1000\ g)(1\ slug/14.5939\ kg)}}{\mathrm{(0.01\ m/cm)(1\ ft/0.3048\ m)s}} = 0.0000209 \frac{\mathrm{slug}}{\mathrm{ft \cdot s}}$$

$$\nu = 0.01 \frac{\mathrm{cm^2}}{\mathrm{s}} = 0.01 \frac{\mathrm{cm^2(0.01\ m/cm)^2(1\ ft/0.3048\ m)^2}}{\mathrm{s}} = 0.0000108 \frac{\mathrm{ft^2}}{\mathrm{s}} \qquad \textit{Ans. (b)}$$

- *Comments:* This was a laborious conversion that could have been shortened by using the direct viscosity conversion factors in App. C or the inside front cover. For example, $\mu_{BG} = \mu_{SI}/47.88$.

We repeat our advice: Faced with data in unusual units, convert them immediately to either SI or BG units because (1) it is more professional and (2) theoretical equations in fluid mechanics are *dimensionally consistent* and require no further conversion factors when these two fundamental unit systems are used, as the following example shows.

EXAMPLE 1.3

A useful theoretical equation for computing the relation between pressure, velocity, and altitude in a steady flow of a nearly inviscid, nearly incompressible fluid with negligible heat transfer and shaft work[5] is the *Bernoulli relation,* named after Daniel Bernoulli, who published a hydrodynamics textbook in 1738:

$$p_0 = p + \tfrac{1}{2}\rho V^2 + \rho g Z \qquad (1)$$

where p_0 = stagnation pressure
p = pressure in moving fluid
V = velocity
ρ = density
Z = altitude
g = gravitational acceleration

(*a*) Show that Eq. (1) satisfies the principle of dimensional homogeneity, which states that all additive terms in a physical equation must have the same dimensions. (*b*) Show that consistent units result without additional conversion factors in SI units. (*c*) Repeat (*b*) for BG units.

[5]That's an awful lot of assumptions, which need further study in Chap. 3.

Solution

Part (a)

We can express Eq. (1) dimensionally, using braces, by entering the dimensions of each term from Table 1.2:

$$\{ML^{-1}T^{-2}\} = \{ML^{-1}T^{-2}\} + \{ML^{-3}\}\{L^2T^{-2}\} + \{ML^{-3}\}\{LT^{-2}\}\{L\}$$

$$= \{ML^{-1}T^{-2}\} \text{ for all terms} \qquad \textit{Ans. (a)}$$

Part (b)

Enter the SI units for each quantity from Table 1.2:

$$\{\mathrm{N/m^2}\} = \{\mathrm{N/m^2}\} + \{\mathrm{kg/m^3}\}\{\mathrm{m^2/s^2}\} + \{\mathrm{kg/m^3}\}\{\mathrm{m/s^2}\}\{\mathrm{m}\}$$

$$= \{\mathrm{N/m^2}\} + \{\mathrm{kg/(m \cdot s^2)}\}$$

The right-hand side looks bad until we remember from Eq. (1.3) that $1 \text{ kg} = 1 \text{ N} \cdot \text{s}^2/\text{m}$.

$$\{\mathrm{kg/(m \cdot s^2)}\} = \frac{\{\mathrm{N \cdot s^2/m}\}}{\{\mathrm{m \cdot s^2}\}} = \{\mathrm{N/m^2}\} \qquad \textit{Ans. (b)}$$

Thus all terms in Bernoulli's equation will have units of pascals, or newtons per square meter, when SI units are used. No conversion factors are needed, which is true of all theoretical equations in fluid mechanics.

Part (c)

Introducing BG units for each term, we have

$$\{\mathrm{lbf/ft^2}\} = \{\mathrm{lbf/ft^2}\} + \{\mathrm{slugs/ft^3}\}\{\mathrm{ft^2/s^2}\} + \{\mathrm{slugs/ft^3}\}\{\mathrm{ft/s^2}\}\{\mathrm{ft}\}$$

$$= \{\mathrm{lbf/ft^2}\} + \{\mathrm{slugs/(ft \cdot s^2)}\}$$

But, from Eq. (1.3), $1 \text{ slug} = 1 \text{ lbf} \cdot \text{s}^2/\text{ft}$, so that

$$\{\mathrm{slugs/(ft \cdot s^2)}\} = \frac{\{\mathrm{lbf \cdot s^2/ft}\}}{\{\mathrm{ft \cdot s^2}\}} = \{\mathrm{lbf/ft^2}\} \qquad \textit{Ans. (c)}$$

All terms have the unit of pounds-force per square foot. No conversion factors are needed in the BG system either.

There is still a tendency in English-speaking countries to use pound-force per square inch as a pressure unit because the numbers are more manageable. For example, standard atmospheric pressure is $14.7 \text{ lbf/in}^2 = 2116 \text{ lbf/ft}^2 = 101{,}300$ Pa. The pascal is a small unit because the newton is less than $\frac{1}{4}$ lbf and a square meter is a very large area.

Consistent Units

Note that not only must all (fluid) mechanics equations be dimensionally homogeneous, one must also use *consistent units;* that is, each additive term must have the same units. There is no trouble doing this with the SI and BG systems, as in Example 1.3, but woe unto those who try to mix colloquial English units. For example, in Chap. 9, we often use the assumption of steady adiabatic compressible gas flow:

$$h + \tfrac{1}{2}V^2 = \text{constant}$$

where h is the fluid enthalpy and $V^2/2$ is its kinetic energy per unit mass. Colloquial thermodynamic tables might list h in units of British thermal units per pound mass (Btu/lb), whereas V is likely used in ft/s. It is completely erroneous to add Btu/lb to ft^2/s^2. The proper unit for h in this case is ft · lbf/slug, which is identical to ft^2/s^2. The conversion factor is 1 Btu/lb $\approx$ 25,040 ft^2/s^2 = 25,040 ft · lbf/slug.

Homogeneous versus Dimensionally Inconsistent Equations

All theoretical equations in mechanics (and in other physical sciences) are *dimensionally homogeneous;* that is, each additive term in the equation has the same dimensions. However, the reader should be warned that many empirical formulas in the engineering literature, arising primarily from correlations of data, are dimensionally inconsistent. Their units cannot be reconciled simply, and some terms may contain hidden variables. An example is the formula that pipe valve manufacturers cite for liquid volume flow rate Q (m^3/s) through a partially open valve:

$$Q = C_V\left(\frac{\Delta p}{\text{SG}}\right)^{1/2}$$

where Δp is the pressure drop across the valve and SG is the specific gravity of the liquid (the ratio of its density to that of water). The quantity C_V is the *valve flow coefficient,* which manufacturers tabulate in their valve brochures. Since SG is dimensionless $\{1\}$, we see that this formula is totally inconsistent, with one side being a flow rate $\{L^3/T\}$ and the other being the square root of a pressure drop $\{M^{1/2}/L^{1/2}T\}$. It follows that C_V must have dimensions, and rather odd ones at that: $\{L^{7/2}/M^{1/2}\}$. Nor is the resolution of this discrepancy clear, although one hint is that the values of C_V in the literature increase nearly as the square of the size of the valve. The presentation of experimental data in homogeneous form is the subject of *dimensional analysis* (Chap. 5). There we shall learn that a homogeneous form for the valve flow relation is

$$Q = C_d A_{\text{opening}}\left(\frac{\Delta p}{\rho}\right)^{1/2}$$

where ρ is the liquid density and A the area of the valve opening. The *discharge coefficient* C_d is dimensionless and changes only moderately with valve size. Please believe—until we establish the fact in Chap. 5—that this latter is a *much* better formulation of the data.

Meanwhile, we conclude that dimensionally inconsistent equations, though they occur in engineering practice, are misleading and vague and even dangerous, in the sense that they are often misused outside their range of applicability.

Table 1.3 Convenient Prefixes for Engineering Units

Multiplicative factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

Convenient Prefixes in Powers of 10

Engineering results often are too small or too large for the common units, with too many zeros one way or the other. For example, to write p = 114,000,000 Pa is long and awkward. Using the prefix "M" to mean 10^6, we convert this to a concise p = 114 MPa (megapascals). Similarly, t = 0.000000003 s is a proofreader's nightmare compared to the equivalent t = 3 ns (nanoseconds). Such prefixes are common and convenient, in both the SI and BG systems. A complete list is given in Table 1.3.

EXAMPLE 1.4

In 1890 Robert Manning, an Irish engineer, proposed the following empirical formula for the average velocity V in uniform flow due to gravity down an open channel (BG units):

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \tag{1}$$

where R = hydraulic radius of channel (Chaps. 6 and 10)
S = channel slope (tangent of angle that bottom makes with horizontal)
n = Manning's roughness factor (Chap. 10)

and n is a constant for a given surface condition for the walls and bottom of the channel. (*a*) Is Manning's formula dimensionally consistent? (*b*) Equation (1) is commonly taken to be valid in BG units with n taken as dimensionless. Rewrite it in SI form.

Solution

- *Assumption:* The channel slope S is the tangent of an angle and is thus a dimensionless ratio with the dimensional notation $\{1\}$—that is, not containing M, L, or T.
- *Approach (a):* Rewrite the dimensions of each term in Manning's equation, using brackets $\{\}$:

$$\{V\} = \left\{\frac{1.49}{n}\right\}\{R^{2/3}\}\{S^{1/2}\} \quad \text{or} \quad \left\{\frac{L}{T}\right\} = \left\{\frac{1.49}{n}\right\}\{L^{2/3}\}\{1\}$$

  This formula is incompatible unless $\{1.49/n\} = \{L^{1/3}/T\}$. If n is dimensionless (and it is never listed with units in textbooks), the number 1.49 must carry the dimensions of $\{L^{1/3}/T\}$. *Ans.* (*a*)

- *Comment (a):* Formulas whose numerical coefficients have units can be disastrous for engineers working in a different system or another fluid. Manning's formula, though popular, is inconsistent both dimensionally and physically and is valid only for water flow with certain wall roughnesses. The effects of water viscosity and density are hidden in the numerical value 1.49.
- *Approach (b):* Part (*a*) showed that 1.49 has dimensions. If the formula is valid in BG units, then it must equal 1.49 $\text{ft}^{1/3}/\text{s}$. By using the SI conversion for length, we obtain

$$(1.49\ \text{ft}^{1/3}/\text{s})(0.3048\ \text{m/ft})^{1/3} = 1.00\ \text{m}^{1/3}/\text{s}$$

  Therefore, Manning's inconsistent formula changes form when converted to the SI system:

$$\text{SI units:}\quad V = \frac{1.0}{n} R^{2/3} S^{1/2} \qquad \textit{Ans. (b)}$$

  with R in meters and V in meters per second.
- *Comment (b):* Actually, we misled you: This is the way Manning, a metric user, first proposed the formula. It was later converted to BG units. Such dimensionally inconsistent formulas are dangerous and should either be reanalyzed or treated as having very limited application.

1.5 Properties of the Velocity Field

In a given flow situation, the determination, by experiment or theory, of the properties of the fluid as a function of position and time is considered to be the *solution* to the problem. In almost all cases, the emphasis is on the space–time distribution of the fluid properties. One rarely keeps track of the actual fate of the specific fluid particles. This treatment of properties as continuum-field functions distinguishes fluid mechanics from solid mechanics, where we are more likely to be interested in the trajectories of individual particles or systems.

The Velocity Field

Foremost among the properties of a flow is the velocity field $\mathbf{V}(x, y, z, t)$. In fact, determining the velocity is often tantamount to solving a flow problem, since other properties follow directly from the velocity field. Chapter 2 is devoted to the calculation of the pressure field once the velocity field is known. Books on heat transfer (for example, Ref. 20) are largely devoted to finding the temperature field from known velocity fields.

In general, velocity is a vector function of position and time and thus has three components u, v, and w, each a scalar field in itself:

$$\boxed{V(x, y, z, t) = \mathbf{i}u(x, y, z, t) + \mathbf{j}v(x, y, z, t) + \mathbf{k}w(x, y, z, t)} \tag{1.4}$$

The use of u, v, and w instead of the more logical component notation V_x, V_y, and V_z is the result of an almost unbreakable custom in fluid mechanics. Much of this textbook, especially Chaps. 4, 7, 8, and 9, is concerned with finding the distribution of the velocity vector $\mathbf{V}$ for a variety of practical flows.

The Acceleration Field

The acceleration vector, $\mathbf{a} = d\mathbf{V}/dt$, occurs in Newton's law for a fluid and thus is very important. In order to follow a particle in the Eulerian frame of reference, the final result for acceleration is nonlinear and quite complicated. Here we only give the formula:

$$\boxed{\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}} \tag{1.5}$$

where (u, v, w) are the velocity components from Eq. (1.4). We shall study this formula in detail in Chap. 4. The last three terms in Eq. (1.5) are nonlinear products and greatly complicate the analysis of general fluid motions, especially viscous flows.

1.6 Thermodynamic Properties of a Fluid

While the velocity field $\mathbf{V}$ is the most important fluid property, it interacts closely with the thermodynamic properties of the fluid. We have already introduced into the discussion the three most common such properties:

1. Pressure p
2. Density ρ
3. Temperature T